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BlackHawk: A public code for calculating the Hawking evaporation spectra of any Black Hole distribution

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Abstract

We describe BlackHawk, a public C program for calculating the Hawking evaporation spectra of any Black Hole distribution. This program enables the users to compute the primary and secondary spectra of stable or long-lived particles generated by Hawking radiation of the distribution of Black Holes, and to study their evolution in time. The physics of Hawking radiation is presented, and the capabilities, features and usage of BlackHawk are described here under the form of a manual.

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1 Introduction

Black Holes (BHs) are fundamental objects which are of utmost importance for the understanding of gravitation. With the detection of gravitational waves from mergers of binary BHs [1–3], direct observation of the Milky Way supermassive central BH [4], and the cosmological and gravitational questions related to primordial BHs (see for example [5–8]), these compact objects are currently under intense scrutiny. It is therefore important to find methods to characterize their properties, and we present here a program for studying multi-messenger probes of BHs.

Other codes, such as `BlackMax` [9] and `Charybdis` [10], have already been released in order to compute the Hawking emission of BHs, which however focus on higher-dimensional models of general relativity in which the Planck mass is decreased, and allow the users to make predictions for generation and evaporation of micro-BHs at high-energy colliders.

We present here `BlackHawk`, which is the first public code for the computation of Hawking evaporation radiation into stable or long-lived particles of 4-dimensional BH mass and spin distributions and its evolution in time.

This document constitutes the manual of `BlackHawk v1.1` and is organized as follows: Section 2 is a brief overview of BHs and Hawking evaporation physics, Section 3 presents the structure and file content of the code, and the compilation and run instructions. Section 4 describes the input parameters needed to run `BlackHawk`, Section 5 gives a detailed description of all the routines written in the code. Section 6 follows the normal execution of `BlackHawk` programs and gives examples of screen output. Section 7 presents the format of the data files generated by a run along with examples, Section 8 gives an estimation of the memory usage and Section 9 provides instructions to the users on how to modify the code.

2 Physics of Hawking evaporation

In this section we give a short overview of the main physical aspects of Hawking evaporation. This concerns BHs of primordial origin (Primordial Black Holes, denoted as PBHs), as well as any other BHs.

In the following, all formulas are in natural units where $\hbar = c = k_{\rm B} = G = 1$, unless stated otherwise.

2.1 Testing BH distributions

`BlackHawk` has been designed to provide tests of compatibility between observations and BH distributions at different main steps of the history of the Universe. For this purpose, it computes the Hawking emission of a distribution of BHs, and its evolution in time. The obtained spectra can then be used to check whether the amount of produced particles has an effect on observable cosmological quantities.

The distribution of BHs as a function of their mass and spin is completely model-dependent and recent studies have proven some previously set constraints to be irrelevant [11]. `BlackHawk` can in principle work with any distribution of BHs. Several BH mass and spin distributions are already built-in and depend on the details of the BHs formation mechanisms.

2.1.1 Peak theory distribution

The peak theory distribution is derived from the scale-invariant model, assuming that the power spectrum of the primordial density fluctuations is a power-law (see for example [12, 13]):

$$P(k) = R_c \left(\frac{k}{k_0}\right)^{n-1} , \tag{2.1}$$

where $n \approx 1.3$ and R_c is measured using the Cosmic Microwave Background (CMB) to be $R_c = (24.0 \pm 1.2) \times 10^{-10}$ at the scale $k_0 = 0.002\,\mathrm{Mpc}^{-1}$. The comoving number density of BHs resulting from this power spectrum is obtained in [12] through peak-theory:

$$\mathrm{d}n \approx \frac{1}{4\pi^2 M} \left(\frac{X(n-1)}{6M}\right)^{3/2} \frac{(n-1)}{2} \nu^4 e^{-\nu^2/2} \mathrm{d}M \,, \tag{2.2}$$

where:

$$\nu(M) = \left(\frac{2(k_0^2 M/X)^{(n-1)/2}}{R_c \Gamma((n-1)/2)}\right)^{1/2} \zeta_{\mathrm{th}} \,, \tag{2.3}$$

and:

$$X = \frac{4\pi}{3} \left(\frac{8\pi G}{3}\right)^{-1} \left(\frac{H_0^2 \Omega_{\mathrm{m}}}{1+z_{\mathrm{eq}}} \left(\frac{g_{*\mathrm{eq}}}{g_*}\right)^{1/3}\right)^{1/2} , \tag{2.4}$$

in which

- $H_0 = 67.8\,\mathrm{km{\cdot}s^{-1}{\cdot}Mpc^{-1}}$ is the current Hubble parameter [14].
- $\Omega_{\mathrm{m}} = 0.308$ is the matter mass fraction in the Universe [14].
- $z_{\mathrm{eq}} = 3200$ is the radiation-matter equality redshift [12].
- $g_{*\mathrm{eq}} = 3.36$ is the number of relativistic energy degrees of freedom (dof) at radiation-matter equality [12].
- $g_* = 106.75$ is the number of relativistic energy dof at the time of BH formation (here the end of the inflation) [14];
- $\zeta_{\mathrm{th}} = 0.7$ parametrizes the direct collapse of a density fluctuation into a BH [12].

2.1.2 Log-normal distribution

The log-normal distribution [11] is considered to be the general mass function originating from a peak in the power spectrum of primordial fluctuations. It is parametrized through:

$$\mathrm{d}n = \frac{A}{\sqrt{2\pi}\sigma M^2}\exp\left(-\frac{\ln(M/M_c)^2}{2\sigma^2}\right)\mathrm{d}M\,, \tag{2.5}$$

where A is the amplitude, M_c is the position of the peak and σ is its width. Note that this is a log-normal distribution for the comoving *density* $M\mathrm{d}n/\mathrm{d}M$ and not for the comoving *number density* $\mathrm{d}n/\mathrm{d}M$ – which differs only by a factor of M:

$$\mathrm{d}n = \frac{A}{\sqrt{2\pi}\sigma M}\exp\left(-\frac{\ln(M/M_c)^2}{2\sigma^2}\right)\mathrm{d}M\,. \tag{2.6}$$

2.1.3 Power-law distribution

The power-law distribution [11] is a less refined version of Eq. (2.2). It also derives from scale-invariant primordial density fluctuations and is given by:

$$\mathrm{d}n = AM^{\gamma-2}\mathrm{d}M\,, \tag{2.7}$$

where $\gamma = -2w/(1+w)$ and w is defined through the equation of state of the dominating energy in the Universe at the epoch of BH formation such as $P = w\rho$ (see Table 1).

fluid	equation of state
matter	$w = 0$
radiation	$w = 1/3$
cosmological constant	$w = -1$
curvature	$w = -1/3$

Table 1. Equation of state for different cosmological fluids.

2.1.4 Critical collapse distribution

The critical collapse distribution [11] derives from a Dirac power spectrum for primordial density fluctuations. It is defined as:

$$\mathrm{d}n = AM^{1.85}\exp\left(-\left(\frac{M}{M_f}\right)^{2.85}\right)\mathrm{d}M\,, \tag{2.8}$$

where A is an amplitude factor and M_f an upper cut-off.

2.2 Uniform distribution

The uniform distribution is a toy-model defined as:

$$\mathrm{d}n = \frac{A}{M_{\max} - M_{\min}}\mathrm{d}M\,, \tag{2.9}$$

where A is an amplitude factor.

2.2.1 Dirac distribution

The Dirac distribution simulates a Dirac BH mass function. It is useful to perform time-dependent monochromatic analyses and checks for a single BH. It is normalized to 1 BH per comoving cm^3.

2.2.2 Spin distributions

Any realistic model of PBH formation should predict a spin distribution as well as a mass distribution. In `BlackHawk`, three kinds of spin distributions are implemented:

- a Dirac distribution,
- a uniform distribution normalized to unity:

$$\mathrm{d}n = \frac{\mathrm{d}a^*}{a^*_{\mathrm{max}} - a^*_{\mathrm{min}}}\,, \tag{2.10}$$

- a Gaussian distribution normalized to unity:

$$\mathrm{d}n = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(a^* - a^*_c)^2}{2\sigma^2}\right) \mathrm{d}a^*\,. \tag{2.11}$$

These spin distributions are independent from the mass distributions, that is to say that e.g. if the mass number density is a Dirac distribution, say $1\,\mathrm{cm}^{-3}$, then this number density can be stretched over an extended spin distribution, say uniform between 0 and 0.5. For joint distributions we recommend that the users use their own distribution (see Section 9.2).

2.3 Hawking evaporation

2.3.1 Schwarzschild Black Holes

Schwarzschild Black Holes are the simplest form of BHs. They are spherically symmetric and only described by their mass M. Hawking has shown [15] that BH horizons emit elementary particles as blackbodies with a temperature linked to their mass M through:

$$T \equiv \frac{1}{8\pi M}\,. \tag{2.12}$$

The number of particles of type i emitted per units of time and energy is:

$$\frac{\mathrm{d}^2 N_i}{\mathrm{d}t\mathrm{d}E} = \sum_{\mathrm{dof}} \frac{\Gamma_i(E,M)/2\pi}{e^{E/T} \pm 1}\,, \tag{2.13}$$

where the sum is over the total multiplicity of the particle (see Table 3 in Appendix C) as well as the angular momentum l of the particle and its projection $m \in \{-l, ..., l\}$ and the $\pm$ are for fermions and bosons, respectively. In the Schwarzschild case, all

angular momentum projections m give the same contribution to Γ_i so this part of the sum results in a $l(l+1)$ factor. The factor Γ_i is called the greybody factor, which is detailed below.

The time-dependent comoving density of Hawking elementary particles i emitted by a distribution of BHs per units of time and energy is then computed through the integral:

$$\frac{\mathrm{d}^2 n_i}{\mathrm{d}t\mathrm{d}E} = \int_{M_{\min}}^{M_{\max}} \int_{a^*_{\min}}^{a^*_{\max}} \frac{\mathrm{d}^2 N_i}{\mathrm{d}t\mathrm{d}E} \cdot \frac{\mathrm{d}^2 n}{\mathrm{d}M\mathrm{d}a^*} \mathrm{d}M\mathrm{d}a^* \,. \tag{2.14}$$

To obtain instantaneous quantities for a single BH of mass M_0 and spin a^*_0, one just needs to take:

$$\frac{\mathrm{d}^2 n}{\mathrm{d}M\mathrm{d}a^*} = \delta(M - M_0)\delta(a - a^*_0) \,. \tag{2.15}$$

The greybody factors describe the probability that a spherical wave representing an elementary particle generated by thermal fluctuations of the vacuum at the BH horizon escapes its gravitational well. We have to solve the Dirac (spin $s = 1/2$) and Proca (integer spin s) wave equations for a particle of rest mass μ:

$$(i\partial\!\!\!/ - \mu)\psi = 0 \,, \tag{2.16}$$

$$(\Box + \mu^2)\phi = 0 \,, \tag{2.17}$$

in the Schwarzschild metric:

$$\mathrm{d}s^2 = h(r)\mathrm{d}t^2 - h(r)^{-1}\mathrm{d}r^2 - r^2(\mathrm{d}\theta^2 + \sin(\theta)^2\mathrm{d}\phi^2) \,, \tag{2.18}$$

where $h(r) \equiv 1 - r_{\mathrm{H}}/r$ and $r_{\mathrm{H}} \equiv 2M$ is the Schwarzschild radius. Teukolsky and Press have shown [16, 17] that the wave equation can be separated into a radial equation and an angular equation if the spherical wave is decomposed into spin weighted spherical harmonics $S_{sl}(\theta)$ and a radial component $R_s(r)$. The radial component of the master equation is, for massless particles of spin s [18]:

$$\frac{1}{\Delta^s}\frac{\mathrm{d}}{\mathrm{d}r}\left(\Delta^{s+1}\frac{\mathrm{d}R_s}{\mathrm{d}r}\right) + \left(\frac{K^2 + is(2r - r_{\mathrm{H}})K}{\Delta} - 4isEr - \lambda_{sl}\right) R_s = 0 \,, \tag{2.19}$$

where $\Delta(r) \equiv r^2 h(r)$, $K(r) \equiv r^2 E^2$ and E is the particle frequency (or equivalently its energy). In this equation, the separation constant $\lambda_{ls} \equiv l(l+1) - s(s+1)$ is the eigenvalue of the angular equation, where l denotes the angular momentum of the spherical harmonics. The main effect of the rest mass μ of particles is to cut the spectra at $E < \mu$ [19], so we set $\mu = 0$ in our computations in order to use this decomposition.

To obtain the greybody factors, one has to compute the transmission coefficients of the wave between the BH horizon and the spatial infinity. The cross-section $\sigma_i(E)$ of the spherical wave on the BH is a sum on all spherical modes l obtained through the optical theorem. The greybody factor is finally given by [20]:

$$\Gamma_i(E, M) = \frac{\sigma_i(E, M)E^2}{\pi} \,. \tag{2.20}$$

The method used in `BlackHawk` to compute those greybody factors is described in Appendix B.1.

2.3.2 Kerr Black Holes

Kerr Black Holes are an extension of the Schwarzschild ones with an additional parameter: their spin $a \equiv J/M \in [0, M]$ (in the following we will denote the reduced spin parameter by $a^* \equiv a/M \in [0, 1]$) where J is the BH angular momentum. These rotating BHs could gain their spin through their formation mechanism [21], accretion [22] or mergers [23]. They are axially symmetric and require a specific treatment.

The temperature of a rotating BH is given by:

$$T \equiv \frac{1}{2\pi}\left(\frac{r_+ - M}{r_+^2 + a^2}\right) , \tag{2.21}$$

where $r_+ \equiv M + \sqrt{M^2 - a^2}$ is the Kerr external radius. The Dirac and Proca equations (2.16) and (2.17) have to be developed in the Kerr metric:

$$\mathrm{d}s^2 = \left(\mathrm{d}t - a\sin^2(\theta)\mathrm{d}\phi\right)^2 \frac{\Delta}{\Sigma} - \left(\frac{\mathrm{d}r^2}{\Delta} + \mathrm{d}\theta^2\right)\Sigma - \left((r^2+a^2)\mathrm{d}\phi - a\mathrm{d}t\right)^2 \frac{\sin^2(\theta)}{\Sigma} , \tag{2.22}$$

where $\Sigma(r) \equiv r^2 + a^2\cos^2(\theta)$ and now $\Delta(r) \equiv r^2 - 2Mr + a^2$. The Teukolsky equation (2.19) is then further modified with $K(r) \equiv (r^2 + a^2)E^2 + am$, where m is the projection of the angular momentum l. The separation constant λ_{slm}, now resulting from the angular solution for spin-weighted spheroidal harmonics $S_{slm}(\theta)$, is more difficult to compute. We will use the 5th order expansion in terms of $\gamma = a^*ME$, as given in [24][4].

The number of particles of type i emitted per units of time and energy is now:

$$\frac{\mathrm{d}^2 N_i}{\mathrm{d}t\mathrm{d}E} = \sum_{\mathrm{dof}} \frac{\Gamma_i(E, M, a^*)/2\pi}{e^{E'/T} \pm 1} , \tag{2.23}$$

where $E' \equiv E - m\Omega$ and $\Omega \equiv a^*/(2r_+)$ is the angular velocity at the horizon [24].

The method used to compute these greybody factors in `BlackHawk` is described in Appendix B.1.

2.3.3 Exotic Black Holes

There are numerous other types of BHs, either in the classical standard cosmological model framework, such as the charged Reissner-Nordström BHs which possess a $U(1)$ electric charge (e.g. [19, 25]), or in alternative models such as (A)dS BHs [26–29], scalar-tensor theories [30–32], higher-dimensional theories [33–37], massive gravity [38, 39], ... These BHs still exhibit a Hawking radiation process in most cases, the two

[4] With our conventions we have an opposite sign for γ as compared to Ref. [24] (all odd-terms in their Appendix A have to be switched in sign).

main differences being Hawking temperature and greybody factors. Equations giving these quantities for specific cases can usually be found in the associated literature. Possible implementations of beyond-standard BHs in `BlackHawk` are described in Section 9.5.

2.4 Black Hole evolution

2.4.1 Schwarzschild Black Holes

Once the greybody factors are known, it is possible to integrate Eq. (2.13) to obtain a differential equation for the mass loss of a BH through Hawking evaporation [40]:

$$\frac{\mathrm{d}M}{\mathrm{d}t} = -\frac{f(M)}{M^2}\,. \tag{2.24}$$

The Page factor $f(M)$ accounts for the number of quantum dof that a BH of mass M can emit. It is obtained through [40]:

$$f(M) = -M^2\frac{\mathrm{d}M}{\mathrm{d}t} = M^2\int_0^{+\infty}\frac{E}{2\pi}\sum_i\sum_{\mathrm{dof}}\frac{\Gamma_i(E,M)}{e^{E/T}\pm 1}\mathrm{d}E\,. \tag{2.25}$$

The computation of the $f(M)$ Page factor in `BlackHawk` is described in Appendix B.2.

2.4.2 Kerr Black Holes

For Kerr BH, a new phenomenon arises; the rotation of the BH enhances the emission of particles with high angular momentum, and with a projection m of that angular momentum aligned with the BH spin, thus effectively extracting angular momentum from the BH. The equation for the Page factor $f(M,a^*)$ becomes [24, 41]:

$$f(M,a^*) = -M^2\frac{\mathrm{d}M}{\mathrm{d}t} = M^2\int_0^{+\infty}\frac{E}{2\pi}\sum_i\sum_{\mathrm{dof}}\frac{\Gamma_i(E,M,a^*)}{e^{E'/T}\pm 1}\mathrm{d}E\,, \tag{2.26}$$

and the differential equation describing the angular momentum J is [24, 41][5]:

$$g(M,a^*) = -\frac{M}{a^*}\frac{\mathrm{d}J}{\mathrm{d}t} = -\frac{M}{a^*}\int_0^{+\infty}\sum_i\sum_{\mathrm{dof}}\frac{m}{2\pi}\frac{\Gamma_i(E,M,a^*)}{e^{E'/T}\pm 1}\mathrm{d}E\,. \tag{2.27}$$

Once the $f(M,a^*)$ and $g(M,a^*)$ Page factors are obtained, the evolution of a^* is straightforwardly obtained through [38]:

$$\frac{\mathrm{d}a^*}{\mathrm{d}t} = \frac{\mathrm{d}(J/M^2)}{\mathrm{d}t} = \frac{1}{M^2}\frac{\mathrm{d}J}{\mathrm{d}t} - 2\frac{J}{M^3}\frac{\mathrm{d}M}{\mathrm{d}t} = a^*\frac{2f(M,a^*)-g(M,a^*)}{M^3}\,. \tag{2.28}$$

The computation of the $f(M,a^*)$ and $g(M,a^*)$ Page factors in `BlackHawk` is described in Appendix B.2.

[5] Same remark as above, we have in our conventions an opposite sign for g.

2.4.3 Exotic Black Holes

Exotic BHs listed in Section 2.3.3 can have a modified evolution as compared to the Schwarzschild and Kerr cases, for two main reasons. First, since their greybody factors and temperature are different, the f and g parameters are expected to be different as well and the master equation (2.23) will give a different emission rate. Second, these BHs can possess other scalar degrees of freedom, such as a $U(1)$ charge (e.g. the electric charge in Reissner-Nordström BHs [19]), which experience a specific evolution. Evolution equations for these additional charges have to be derived, and would be similar to Eqs. (2.26) and (2.27). Possible implementations of beyond-standard BHs in `BlackHawk` are described in Section 9.5.

2.5 Hadronization

The elementary particles emitted by BHs are not the final products of the Hawking emission. Some of them are unstable, others only exist in hadrons. A particle physics code has to be used in order to evolve the elementary particles into final products. We used `HERWIG` [42] and `PYTHIA` [43] for this purpose.

The final particles, hereby denoted as "secondary Hawking particles" (the elementary being the "primary Hawking particles"), depend on the cosmological context in which they are emitted. For Big-Bang Nucleosynthesis (BBN) studies, an estimation of the reaction rates imposes to keep the particles with a lifetime longer than $\sim 10^{-8}\,$s. These particles are listed in the Table 3 of Appendix C.

The time-dependent comoving density of Hawking secondary particle j emitted by a distribution of BHs per units of time and energy is computed with the integral:

$$\frac{\mathrm{d}^2 n_j}{\mathrm{d}t\mathrm{d}E} = \int \sum_i \frac{\mathrm{d}^2 n_i}{\mathrm{d}t\mathrm{d}E'} \cdot \frac{\mathrm{d}N_j^i}{\mathrm{d}E}\,\mathrm{d}E'\,, \tag{2.29}$$

where the sum is taken over Hawking primary particles i, and Appendix B.3 describes how hadronization tables $\mathrm{d}N_j^i(E', E)$ have been computed to transform the primary spectra into secondary spectra in `BlackHawk`.

3 Content and compilation

This section describes the structure and file content of the code and explains its usage. `BlackHawk` is written in C and has been tested under Linux, Mac and Windows (using `Cygwin64`).

3.1 Main directory

The main directory contains:

- the source codes `BlackHawk_inst.c` and `BlackHawk_tot.c` containing the `main` routines,

- a pre-built parameter file `parameters.txt`,
- a compilation file `Makefile`,
- a `README.txt` file containing general information about the code,
- four folders `src/`, `results/`, `manual/` and `scripts/` that are described in the following.

3.2 src/ subfolder

This folder contains:

- a header file `include.h` containing the declaration of all routines along with the parameter structure `struct param` (see Section 4.1) and the numerical values of general quantities (units conversion factors, constants, particle masses...),
- 9 source files containing the definition of all the `BlackHawk` routines (`evolution.c`, `general.c`, `hadro_herwig.c`, `hadro_pythia.c`, `hadro_pythianew.c`, `primary.c`, `secondary.c`, `spectrum.c`, `technical.c`),
- 2 compilation files `Makefile` and `FlagsForMake`,
- a subfolder `tables/` containing all the numerical tables which will be described in the following.

3.3 results/ subfolder

This folder is designed to receive subfolders of data generated by running the `BlackHawk` code (see Section 7).

3.4 manual/ subfolder

This folder contains an up-to-date version of the present manual.

3.5 scripts/ subfolder

This folder contains all the scripts used to compute the numerical tables mentioned in the following, as well as visualization scripts and a main program for `SuperIso Relic` [44–46]. These scripts can be used to generate modified numerical tables for example (see Section 9 and Appendix B). They are accompanied by `README.txt` files explaining how to use them.

3.6 Compilation

The compilation of BlackHawk has been tested on Linux, Mac and Windows (using Cygwin64) distributions. The code is written in C99 standard. To compile the code, simply cd into the main directory and type[6]:

```
make BlackHawk_*
```

where * denotes tot or inst. This will create a library file libblackhawk.a and an executable file BlackHawk_*.x. The compiler and compilation flags can be modified in Makefile if needed.

To run the code, cd to the main directory and type[7]:

```
./BlackHawk_*.x parameter_file
```

where parameter_file is the name of a parameter file (e.g. parameters.txt for the pre-built one). To compile only the library, just cd into the main directory and type:

```
make
```

4 Input parameters

In this section we describe how input parameters are handled in BlackHawk and their meaning.

4.1 Parameter structure

The input parameters used by BlackHawk are listed in a parameter file (e.g. parameters.txt for the pre-built one). This file can be modified by the user and is saved for each new run of the code in the results/ directory. A C structure has been defined in include.h to embed all the parameters (i.e. input parameters and default run parameters):

```
struct param {
    char destination_folder[32];
    int full_output;
    int interpolation_method;

    int BHnumber;
```

[6] In case of problems of memory size at compilation, editing src/include.h and commenting #define HARDTABLES can solve the problem at the price of a longer execution time.

[7] In case of memory problem at execution, increasing the stack size with the command ulimit -s unlimited can help solving the problem.

```
double Mmin;
double Mmax;
int anumber;
double amin;
double amax;
int spectrum_choice;
int spectrum_choice_a;
double amplitude;
double stand_dev;
double crit_mass;
double eqstate;
double stand_dev_a;
double mean_a;
char table[32];

double tmin;
int nb_fin_times;
int limit;
double Mmin_fM;
double Mmax_fM;
double amin_fM;
double amax_fM;
int nb_fM_masses;
int nb_fM_a;

int Enumber;
double Emin;
double Emax;
int particle_number;
int grav;
int nb_gamma_a;
int nb_gamma_x;

int primary_only;
int hadronization_choice;
double Emin_hadro;
double Emax_hadro;
int nb_init_en;
int nb_fin_en;
int nb_init_part;
int nb_fin_part;
```

};

Most routines described in Section 5 will use this structure as an argument in order to have an easy access to the run parameters. Depending on the choices of the parameters, some of them can be irrelevant for a given run and will therefore not be taken into account, and no error message will be displayed for the irrelevant/unused parameters.

4.2 General parameters

The first set of parameters defines the general variables:

- `destination_folder` is the name of the output folder that will be created in `results/` to save run data.
- `full_output` determines whether the shell output will be expanded (`full_output = 1`) or not (`full_output = 0`). It can be useful for debugging the code or seeing the progress in time-consuming routines. It also determines whether some interactive pre-run checks are done with the user. We thus recommend that this parameter is always set to 1.
- `interpolation_method` determines whether the interpolations in the tables are made linearly (interpolation between the tabulated values) or logarithmically (linear interpolation between the decimal logarithm of the tabulated values).

4.3 BH spectrum parameters

The second set of parameters defines the quantities used to compute the BH density distribution (see Section 5.2):

- `BHnumber` is the number of BH masses that will be simulated. If the parameter `spectrum_choice` is not set to -1, it has to be an integer greater than or equal to 1. If it is equal to 0, the only BH mass will be `Mmin` (see below). If the parameter `spectrum_choice` is set to -1, it has to be the number of tabulated values in the user-defined BH distribution (see below and Section 9.2). It will be automatically set to 1 if `spectrum_choice` is set to 0.
- `Mmin` and `Mmax` are respectively the lowest and highest BH masses that will be simulated. They have to be given in grams and satisfy the condition $M_{\rm p} \approx 2 \times 10^{-5}\,{\rm g} <$ `Mmin`, `Mmax`, where $M_{\rm p}$ is the Planck mass. For an extended mass distribution, one must have `Mmin` $<$ `Mmax`. If they are not compatible with boundaries of the mass distribution, the computation will stop (see below).
- `anumber` is the number of BH spins that will be simulated. If the parameter `spectrum_choice` is not set to -1, it has to be an integer greater than or

equal to 1. If it is equal to 0, the only BH spin will be `amin` (see below). If the parameter `spectrum_choice` is set to -1, it has to be the number of tabulated values in the user-defined BH distribution (see below and Section 9.2). It will be automatically set to 1 if `spectrum_choice_a` is set to 0.

- `amin` and `amax` are respectively the lowest and highest BH dimensionless reduced spins that will be simulated. They have to satisfy the condition $0 \leq$ `amin`, `amax` < 1. For an extended spin distribution, one must have `amin` $<$ `amax`. If they are not compatible with boundaries of the spin distribution, the computation will stop (see below).

- `spectrum_choice` selects the form of the BH mass distribution (see Section 2.1). It has to be an integer among 0 (Dirac), 1 (log-normal for the mass density), 11 (log-normal for the number density), 2 (power-law), 3 (critical collapse), 4 (peak theory), 5 (uniform) and -1 (user-defined distribution, see below and Section 9.2).

- `spectrum_choice_a` selects the form of the BH spin distribution (see Section 2.1). It has to be an integer among 0 (Dirac), 1 (uniform) and 2 (Gaussian). If the parameter `spectrum_choice` has been set to -1, then the spin distribution is also the user-defined one and this parameter is irrelevant.

- `amplitude` is the amplitude A present in Eqs. (2.5), (2.6), (2.7), (2.8) and (2.9). It is the normalization of the corresponding BH distribution and thus strictly positive, its unit depends on the distribution chosen, but masses are in g and densities in cm^{-3}.

- `stand_dev` is the dimensionless standard deviation σ in the log-normal distributions of Eqs. (2.5) and (2.6). It has to be strictly positive.

- `crit_mass` is the characteristic mass M_c in Eqs. (2.5) and (2.6) and M_f in Eq. (2.8). It has to be strictly positive and given in grams.

- `eq_state` defines the equation of state w (see Section 2.1.3), it is dimensionless.

- `stand_dev_a` is the dimensionless standard deviation σ in the gaussian distribution of Eq. (2.11). It has to be strictly positive.

- `mean_a` is the characteristic dimensionless reduced spin a_c^* in the gaussian distribution of Eq. (2.11). It has to be strictly positive.

- `table` is the name of a user-defined BH distribution table. It has to be a string with any file extension.

If the parameters `spectrum_choice` and `spectrum_choice_a` are both set to 0, then the distribution is monochromatic in both mass and spin, mimicking a single BH – that is to say, the emissivities obtained are those of a single BH. This option can be useful to compute and compare known test-emissivities of single BHs.

4.4 BH evolution parameters

The next set of parameters defines the quantities used to compute the BH evolution (see Section 5.3):

- `tmin` is the initial integration time of the evolution of BH, in seconds. It can have any positive value.
- `nb_fin_times` is the number of final integration times that will be used in the computations. It will be set automatically by the integration procedure.
- `limit` is the iteration limit when computing the time evolution of a single BH (see Section 5.3). It is fixed to `limit = 5000` even if the effective iteration numbers hardly reach 1000. It should be increased if the integration does not reach the complete evaporation of BHs or if the error:

  ```
  [life_evolution] :  ERROR ITERATION LIMIT REACHED !
  ```

  appears.
- `Mmin_fM`, `Mmax_fM`, `amin_fM`, `amax_fM`, `nb_fM_masses` and `nb_fM_a` are the BH mass and spin boundaries and numbers (respectively) used to compute the $f(M, a^*)$ and $g(M, a^*)$ tables. They are read by `BlackHawk` in the table information files (see Section 5.1).

4.5 Primary spectrum parameters

This set of parameters defines the quantities related to the primary Hawking spectra (see Section 5.4):

- `Emin` and `Emax` are the minimum and maximum primary particle energies, respectively. They must be compatible with the table boundaries (see below) and satisfy $0 <$ `Emin` $<$ `Emax`.
- `Enumber` is the number of primary particle energies that will be simulated. It has to be an integer greater than or equal to 2.
- `particle_number` is the number of primary particle types. It is fixed to 15 (photon, gluon, $W^{\pm}$ boson, Z^0 boson, Higgs boson, neutrino, 3 leptons (electron, muon, tau) and 6 quarks (up, down, charm, strange, top, bottom)), and `nb_gamma_a` and `nb_gamma_x` are respectively the number of spins and $x \equiv 2 \times E \times M$ tabulated in the greybody factor tables. They are read by `BlackHawk` in the table information files (see Section 5.1).

- `grav` determines whether the emission of gravitons by BH will be taken into account (`grav = 1`) or not (`grav = 0`).

4.6 Hadronization parameters

This last set of parameters defines the quantities used during hadronization (see Section 5.5):

- `primary_only` determines whether the secondary spectra will be computed or not. It has to be an integer between 0 (primary spectra only) and 1 (primary and secondary spectra). In the case where the parameters `Emin` and `Emax` are not compatible with the hadronization table boundaries (see below), a warning will be displayed and extrapolation used.

- `hadronization_choice` determines which hadronization tables will be used to compute the secondary spectra (see Section B.3). It has to be an integer between 0 (`PYTHIA` tables – early Universe/BBN epoch), 1 (`HERWIG` tables – early Universe/BBN epoch) and 2 (new `PYTHIA` tables – present epoch).

- `Emin_hadro` and `Emax_hadro` are the energy boundaries of the hadronization tables, `nb_init_en` and `nb_fin_en` are the number of initial and final particle energy entries and `nb_init_part` and `nb_fin_part` are the number of primary and secondary particle types. They are read by `BlackHawk` in the table information files (see Section 5.1).

5 Routines

Below are listed the main routines defined in `BlackHawk`. To simplify the analytic formulas, all intermediate quantities are in GeV (see Appendix A for conversion rules).

5.1 General routines

There are seven general routines in the `BlackHawk` code. The principal ones are the `main` routines, described in Section 6. The other five are:

- `int read_params(struct param *parameters, char name[], int session)`: this routine reads the file `name` and the tables information files thanks to the `read_*_infos` routines. The parameters are converted from CGS units to GeV. The user should respect the original syntax when modifying the parameters (concerning spaces, underscores, ...), except for comments which are preceded by a `#` symbol. It takes a pointer to a `struct param` object (see Section 4.1) as an argument and fills it using the file `name`. The argument `session` keeps track of which one of the main programs has been launched (0

for `BlackHawk_tot`, 1 for `BlackHawk_inst`). If one parameter is not of the type described in Section 4 this function will display an error message. Any of these errors will return the value 0 and end the `BlackHawk` run. If one parameter is in small contradiction with the others but the computation can still be partly done (e.g. only the primary spectra can be computed with the given parameters) a warning message will be displayed. In such case, the problematic parameters will be set automatically (e.g. `primary_only = 1`) and the computation will be performed.

- `int read_fM_infos(struct param *parameters)`: this routine reads the $f(M, a^*)$ and $g(M, a^*)$ table information in the file:

  `src/tables/fM_tables/infos.txt`

  It returns 1 if all parameters are correct and 0 if there is an error, displaying a message in this case.

- `int read_gamma_infos(struct param *parameters)`: this routine reads the greybody factor table information in the file:

  `src/tables/gamma_tables/infos.txt`

  It returns 1 if all parameters are correct and 0 if there is an error, displaying a message in this case.

- `int read_hadronization_infos(struct param *parameters)`: this routine reads the $f(M, a^*)$ and $g(M, a^*)$ table information in the file:

  `src/tables/*_tables/infos.txt`

  depending of the `hadronization_choice`. It returns 1 if all parameters are correct and 0 if there is an error, displaying a message in this case.

- `int memory_estimation(struct param *parameters, int session)`: this routine gives a rough estimate of the usage of both RAM and disk space (see Section 8). If the user decides to cancel the run the value 0 is returned, otherwise it is 1. The output is given in MB.

5.2 BH spectrum routines

There are six routines contributing to the BH initial spectrum computation (see Section 2.1):

- `void read_users_table(double *init_masses, double *init_spins, double **spec_table, struct param *parameters)`: this routine reads a user-defined BH distribution table in the file given by the parameter `table`. It fills the arrays `init_masses[]`, `init_spins[]` and `spec_table[][]` with results converted from CGS units to GeV.

- `double nu(double M)`: this routine takes a BH mass as an argument and computes the dimensionless quantity $\nu(M)$ defined in Eq. (2.3).

- `double M_dist(double M, struct param *parameters)`: this routine takes a BH mass as an argument and computes the comoving density $\frac{\mathrm{d}n}{\mathrm{d}M}$ defined in Eq. (2.2) (using the `nu` routine) or (2.5) or (2.7) or (2.8) or (2.9) (in $\mathrm{GeV}^2 \to \mathrm{cm}^{-3}{\cdot}\mathrm{g}^{-1}$), depending on the parameter `spectrum_choice` (see Section 4.3). If this parameter is set to 0, a flat distribution is used with only one BH mass.

- `double a_dist(double a, struct param *parameters)`: this routine takes a BH spin as an argument and computes the fraction $\frac{\mathrm{d}n}{\mathrm{d}a}$ defined in Eq. (2.10) or (2.11) (dimensionless), depending on the parameter `spectrum_choice_a` (see Section 4.3). If this parameter is set to 0, a flat distribution is used with only one BH spin.

- `void spectrum(double *init_masses, double *init_spins, double **spec_table, struct param *parameters)`: this routine fills the array `init_masses[]` with `BHnumber` BH masses logarithmically distributed between `Mmin` and `Mmax`. If the parameter `BHnumber` is set to 1, the only BH initial mass will be `Mmin`. It then fills the array `init_spins[]` with spins linearly distributed and finally fills the array `spec_tables[][]` computing the corresponding comoving densities $\mathrm{d}n(M, a^*)$ (in $\mathrm{GeV}^3 \to \mathrm{cm}^{-3}$) using the `M_dist` and `a_dist` routines where $\mathrm{d}M$ is taken around the considered mass and $\mathrm{d}a$ around the considered spin. The result is rescaled by a factor 10^{100} due to the very small numbers involved.

- `void write_spectrum(double *init_masses, double *init_spins, double **spec_table, struct param *parameters)`: this routine writes the BH initial masses, spins and comoving densities in a file `BH_spectrum.txt`, saved in `destination_folder/` (see Section 7.1). The results are converted from GeV to CGS units.

5.3 BH evolution routines

There are ten routines contributing to the BH time evolution computation (see Section 2.4):

- `double rplus_BH(double M, double a)`: this routine gives the external Kerr radius of a rotating BH for a given mass M and reduced spin a^* (see Section 2.3.2) (in $\mathrm{GeV}^{-1} \to \mathrm{cm}$);

- `double temp_BH(double M, double a)`: this routine gives the Hawking temperature of a Kerr BH for a given mass M and reduced spin a^* using Eq. (2.21) (in $\mathrm{GeV} \to \mathrm{K}$).

- `void read_fM_table(double **fM_table, double *fM_masses, double *fM_a, struct param *parameters)`: this routine reads the $f(M, a^*)$ factor (see Eq. (2.26)) in the table contained in the folder `fM_tables/` (see Section B.2). It fills the arrays `fM_masses[]` (in GeV $\rightarrow$ g), `fM_a[]` and `fM_table[][]` with format [mass][spin] (in $\text{GeV}^4 \rightarrow \text{g}^3{\cdot}\text{s}^{-1}$).

- `void read_gM_table(double **gM_table, double *fM_masses, double *fM_a, struct param *parameters)`: this routine reads the $g(M, a^*)$ factor (see Eq. (2.27)) in the table contained in the folder `fM_tables/` (see Section B.2). It fills the arrays `fM_masses[]` (in GeV $\rightarrow$ g), `fM_a[]` and `gM_table[][]` with format [mass][spin] (in $\text{GeV}^4 \rightarrow \text{g}^2{\cdot}\text{GeV}{\cdot}\text{s}^{-1}$).

- `double loss_rate_M(double M, double a, double **fM_table, double *fM_masses, double *fM_a, int counter_M, int counter_a, struct param *parameters)`: this routine computes the quantity $\dfrac{\mathrm{d}M}{\mathrm{d}t}$ defined in Eq. (2.26) (in $\text{GeV}^2 \rightarrow \text{g}{\cdot}\text{s}^{-1}$).

- `double loss_rate_a(double M, double a, double **fM_table, double **gM_table, double *fM_masses, double *fM_a, int counter_M, int counter_a, struct param *parameters)`: this routine computes the quantity $\dfrac{\mathrm{d}a^*}{\mathrm{d}t}$ defined in Eq. (2.28) (in GeV $\rightarrow \text{s}^{-1}$).

- `evolution_times(double *init_masses, double *init_spins, double **evol_times, double **fM_table, double **gM_table, double *fM_masses, double *fM_a, struct param *parameters)`: this routine computes the lifetime of all BHs with masses in `init_masses[]` and spins in `init_spins[]` and stores the results in `evol_times[][]` with format [mass][spin] (in $\text{GeV}^{-1} \rightarrow$ s).

- `sort_lifetimes(double **evol_times, double *sorted_times, int **rank, struct param *parameters)`: this routine sorts the lifetimes of the BHs computed by `evolution_times` from shortest to longest using the routine `sort_fusion` and stores the corresponding ranks in the table `rank[][]` with format [mass][spin].

- `void life_evolution(double ***life_masses, double ***life_spins, double *life_times, double *dts, double *init_masses, double *init_spins, int **rank, double **fM_table, double **gM_table, double *fM_masses, double *fM_a, struct param *parameters)`: this routine computes the evolution of each of the initial BH masses in `init_masses[]` and BH spins in `init_spins[]`. The initial time `life_times[0]` is set to `tmin`, the initial masses `life_masses[i][j][0]` are set to `init_`

`masses[i]` and the initial spins `life_spins[i][j][0]` are set to `init_spins[j]`. Iteratively, the next masses and spins are estimated using the Euler method:

$$M(t + \mathrm{d}t) = M(t) + \frac{\mathrm{d}M}{\mathrm{d}t}\mathrm{d}t\,, \tag{5.1}$$

$$a^*(t + \mathrm{d}t) = a^*(t) + \frac{\mathrm{d}a^*}{\mathrm{d}t}\mathrm{d}t\,, \tag{5.2}$$

where the derivatives are computed using the `loss_rate_*` routines. If the mass or the spin relative variations of the currently interesting BH are too large ($|\mathrm{d}X/X| > 0.1$) then the time interval is divided by 2. If all the variations are very small ($|\mathrm{d}X/X| < 0.001$), and if the current timestep is reasonable compared to the current timescale ($\mathrm{d}t/t \lesssim 1$) then the time interval is multiplied by 2. Once the dimensionless spin reaches 10^{-3}, we stop computing its variation and simply set it to 0, and it does not enter anymore in the adaptive timesteps conditions. This goes on until each BH reaches the Planck mass. The interesting BH is determined by following the order given by the array `rank[][]` which contains the information about the order in which the BHs will evaporate. If the recursion limit `limit` $\times$ `BHnumber` is reached, the following error is displayed:

```
[life_evolution] :  ERROR ITERATION LIMIT REACHED !
```

This may be a sign that the parameter `limit` should be increased. The intermediate time intervals $\mathrm{d}t$, times t, masses M and spins a^* are stored in the arrays `dts[]`, `life_times[]` (both in $\mathrm{GeV}^{-1} \to \mathrm{s}$), `life_masses[][][]` (in $\mathrm{GeV} \to \mathrm{g}$) and `life_spins[][][]`, respectively.

- `void write_life_evolutions(double ***life_masses, double ***life_spins, double *life_times, struct param *parameters)`: this routine writes the BH time-dependent masses and spins until full evaporation in the file `life_evolutions.txt`, saved in `destination_folder/` (see Section 7.1). If the BH distribution is extended in both mass and spin, we advise the user to deactivate this writing because of the extensive memory space use (see Section 8). The results are converted from GeV to CGS units.

5.4 Primary spectra routines

There are five routines contributing to the computation of the primary Hawking spectra (see Section 2.3):

- `void read_gamma_tables(double ***gammas, double *gamma_a, double *gamma_x, struct param *parameters)`: this routine reads the quantities

$\Gamma_i/(e^{E'/T} \pm 1)$, defined in Eq. (2.23), in the tables `spin_*.txt` in the folder `gamma_tables/`. It fills the arrays `gamma_a[]` and `gamma_x[]` with the tabulated spins a^* (dimensionless) and $x \equiv Er_{\rm BH}$ (dimensionless $\rightarrow$ GeV·cm), respectively. It fills the array `gammas[][][]` with the corresponding dimensionless greybody factors in format [type][spin][x] (see Appendix B.1).

- `void read_asymp_fits(double ***fits, struct param *parameters)`: this routine reads the asymptotic fit parameters for the greybody factors, contained in the tables `spin_*_fits.txt` in the folder `gamma_tables/`. It fills the array `fits[][][]` in format [type][spin][parameters] (see Appendix B.1).

- `double dNdtdE(double E, double M, double a, int particle_index, double ***gammas, double *gamma_a, double *gamma_x, double ***fits, double *dof, double *spins, double *masses_primary, int counter_a, int counter_x, struct param *parameters)`: this routine computes the emission rate $\mathrm{d}^2N_i/\mathrm{d}t\mathrm{d}E$ of the primary particle `particle_index` (see Eq. (2.23)), for a given particle energy `E`, the BH mass `M`, the BH reduced spin `a` and the particle informations contained in `dof[]`, `spins[]` and `masses_primary[]`. If $x \equiv Er_{\rm H}$ is between the greybody factor tables boundaries, the values are interpolated in those tables at position `counter_a` and `counter_x`. Otherwise, we use the asymptotic high- and low-energy fits tables (see Appendix B.1). The result is dimensionless ($\rightarrow$ GeV^{-1}·s^{-1}).

- `void instantaneous_primary_spectrum(double **instantaneous_primary_spectra, double *BH_masses, double *BH_spins, double **spec_table, double *energies, double ***gammas, double *gamma_a, double *gamma_x, double ***fits, double *dof, double *spins, double *masses_primary, struct param *parameters)`: this routine computes the instantaneous primary Hawking spectra for a distribution of BHs given by the routine `spectrum`, namely the quantities $\dfrac{\mathrm{d}^2n_i}{\mathrm{d}t\mathrm{d}E}$ in Eq. (2.14) for each primary particle i and each energy in `energies[]`, computed with the routine `dNdtdE`. The results are stored in the array `instantaneous_primary_spectra[][]` with format [particle][energy].

- `void write_instantaneous_primary_spectra(double **instantaneous_primary_spectra, double *energies, struct param *parameters)`: this routine writes the instantaneous primary Hawking spectra in a file `instantaneous_primary_spectra.txt`, saved in `destination_folder/` (see Section 7.2). The results are converted from GeV to CGS units.

5.5 Secondary spectrum routines

There are twelve routines contributing to the computation of the secondary Hawking spectra (see Section 2.5):

- `void convert_hadronization_tables(double ****tables, double *initial_energies, double *final_energies, struct param *parameters)`: this routine is auxiliary. It writes hardcoded versions of the hadronization tables (see Appendix B.3) in files `hadronization_tables_*.h` in the `tables/` subfolder in order to accelerate the code execution, while slowing its compilation.

- `void read_hadronization_tables(double ****tables, double *initial_energies, double *final_energies, struct param *parameters)`: this routine reads the hadronization table (see Appendix B.3) determined by `hadronization_choice`. If `HARDTABLES` is defined, it uses the table included at compilation thanks to the routines `read_hadronization_*`, otherwise it reads the corresponding table in the `tables` subfolder. It fills the arrays `initial_energies[]` and `final_energies[]` with the tabulated primary particles and secondary particles energies (in GeV), respectively, and fills the array `tables[][][][]` with the corresponding branching ratios $\frac{\mathrm{d}N_j^i}{\mathrm{d}E'}$ in Eq. (2.29) (in GeV^{-1}) with format [secondary particle][initial energy][final energy][primary particle].

- `void total_spectra(double ***partial_hadronized_spectra, double **partial_primary_spectra, double **partial_integrated_hadronized_spectra, double ****tables, double *initial_energies, double *final_energies, double ***primary_spectra, double *times, double *energies, double *masses_secondary, struct param *parameters)`: this routine is a container that uses the "instantaneous" routines to compute the Hawking primary and secondary spectra at each steptime in `times` and writes it directly in the output in order to save RAM memory. To do so, it creates the output files `*_primary_spectrum.txt` and `*_secondary_spectrum.txt` (if `primary_only` is set to 0). It reads the writing instructions using `read_writing_instructions`. Then, it fills the partial arrays `partial_*` with the instantaneous primary spectra, hadronized spectra and integrated spectra at each intermediate time and calls the routine `write_lines` to write the partial result in the output before moving to the next time step.

- `void read_writing_instructions(int *write_primary, int *write_secondary, struct param *parameters)`: this routine reads the writing informations contained in the files `src/tables/write_*.txt` (depending

on the `hadronization_choice`) and stores it into the arrays `write_primary[]` and `write_secondary[]`.

- `void write_lines(char **file_names, double **partial_integrated_hadronized_spectra, int *write_primary, int *write_secondary, double time, struct param *parameters)`: given a `time` and instantaneous primary and secondary spectra (if `primary_only` is set to 0), this routine writes a new line in the `*_primary_spectrum.txt` and `*_secondary_spectrum.txt` files. The arrays `write_*[]` determine whether the values for each particles are written or not, thus potentially saving static memory. Results are converted from GeV to CGS units (see Section 7.1).

- `double contribution_instantaneous(int j, int counter, int k, double **instantaneous_primary_spectra, double ****tables, double *initial_energies, double *final_energies, int particle_type, int hadronization_choice)`: this routine computes the instantaneous integrand of Eq. (2.29) (in $\text{GeV}^{-1} \to \text{GeV}^{-2}\cdot\text{s}^{-1}$) for the secondary particle `particle_type`, initial energy $E' =$ `energies[j]`, corresponding tabulated initial energy `initial_energies[counter]` and final energy $E =$ `final_energies[k]`. The sum over channels of production of the secondary particles may depend on the structure of the hadronization tables.

- `void hadronize_instantaneous(double ***instantaneous_hadronized_spectra, double ****tables, double *initial_energies, double *final_energies, double **instantaneous_primary_spectra, double *energies, struct param *parameters)`: this routine computes the instantaneous secondary Hawking spectra for all secondary particles, all initial energies in `energies[]` and all final energies in `final_energies[]`. It fills the array `instantaneous_hadronized_spectra[][][]` using the routine `contribution_instantaneous`, with format [secondary particle][initial energy][final energy]. If the initial energy is not in the hadronization tables, the contribution is extrapolated.

- `void integrate_initial_energies_instantaneous(double ***hadronized_emission_spectra, double **integrated_hadronized_spectra, double *energies, double *final_energies, struct param *parameters)`: this routine computes the integral Eq. (2.29) (dimensionless $\to$ $\text{GeV}^{-1}\cdot\text{s}^{-1}$) using the `trapeze` routine. The results are stored in the array `instantaneous_integrated_hadronized_spectra[][]` with format [secondary particle][final energy].

- `void add_*_instantaneous(double **instantaneous_primary_spectra, double **instantaneous_integrated_hadronized_spectra, double`

`*energies, double *final_energies, struct param *parameters)`: these three routines add the contributions of the primary photons, neutrinos and electrons to the secondary produced ones. The value in term of final energies is interpolated in the primary spectrum and added to the hadronized spectrum `instantaneous_integrated_hadronized_spectra[][]`.

- `void write_instantaneous_hadronized_spectra(double **instantaneous_integrated_hadronized_spectra, double *hadronized_energies, struct param *parameters)`: this routine writes the instantaneous secondary Hawking spectra in the file `instantaneous_secondary_spectra.txt`, saved in `destination_folder/` (see Section 7.2). The results are converted from GeV to CGS units.

5.6 Auxiliary routines

Eight auxiliary routines are used throughout the code:

- `double trapeze(double x1, double x2, double y1, double y2)`: this routine performs the trapeze integration of a function f that takes values `y1` in `x1` and `y2` in `x2` using:

$$\int_{\mathtt{x1}}^{\mathtt{x2}} f(x)\mathrm{d}x \approx \frac{1}{2}(\mathtt{x2}-\mathtt{x1})\times(\mathtt{y1}+\mathtt{y2})\,.$$

- `void free1D(double *array)`, `void free2D_int(int **array, int l_1stD)`, `void free2D_double(double **array, int l_1stD)`, `void free2D_char(char **array, int l_1stD)`, `void free3D_double(double ***array, int l_1stD, int l_2ndD)`, `void free4D_double(double ****array, int l_1stD, int l_2ndD, int l_3rdD)`: these routines perform a proper memory freeing of $n-$dimensional arrays of various types, by recursively applying the native `free` routine.

- `int ind_max(double *table, int llength)`: this routine returns the index of the maximum of the array `table[]` of length `llength`.

- `void fusion(double *table, int start1, int end1, int end2)`, `void sort_fusion_bis(double *table, int start, int end)` and `void sort_fusion(double *table, int llength)`: these 3 routines perform a fusion sorting of the table `table` of length `llength`.

5.7 General features

We have defined arrays `compute[]` that contain either 0's or 1's for all the primary and secondary particles and decide whether the contribution of the corresponding

particle will be taken into account in the computation of the Hawking spectra. For example, if a "0" is set for the primary photon computation, then its primary spectrum will be all 0's and thus will not contribute to the secondary spectra. If a "0" is set for the secondary photon computation, then the secondary particles generated by primary photons are not taken into account in the secondary spectra (but the primary spectra of photons is still computed). These arrays are by default all 1's and should be modified only if the user wants to test specific scenarios or correct a bug.

6 Programs

The `BlackHawk` code is split into two programs, which are presented in this section:

- `BlackHawk_tot`: full time-dependent Hawking spectra;
- `BlackHawk_inst`: instantaneous Hawking spectra.

Once a set of parameters is chosen, the two programs can be launched in the same `destination_folder/` because the output files will not enter in conflict (see Section 7). We will now describe the structure of the `main` routines together with screen output examples.

6.1 Common features

When running the `BlackHawk` code, some routines will be called regardless of the program choice. First, some general quantities are fixed (which are converted into GeV when applicable, see Appendix A):

- `machine_precision` $= 10^{-10}$ defines the precision up to which two `double` numbers are considered as equal.
- `G` $= 6.67408 \times 10^{-11}\,\mathrm{m}^3\cdot\mathrm{kg}^{-1}\cdot\mathrm{s}^{-2}$ is the Newton constant in SI units.
- `Mp` $\equiv G^{-1/2}$ is the Planck mass in the natural system of units.
- `m_*` are the masses of the Standard Model elementary and composite particles (see Table 3 in Appendix C).
- `*_conversion` are the quantities used to convert units from CGS/SI to GeV (see Appendix A).

The code works in several steps, which are separated on the output screen. A new step starts with:

```
[main] :  ***** ...
```

and ends with:

```
                                                                    DONE
```

If the `full_output` parameter is set to 1, then more information will be displayed about the progress of the steps and an interactive pre-run check will be performed. In the case where information appears with the name of another routine inside brackets, it means that an error occurred.

The first common step is the definition and filling of the parameters structure using `read_params`. Then if `full_output` is set to 1 an estimation of the memory that will be used is displayed by `memory_estimation`. The user can choose to go on or to cancel the run (see Section 5.1). If no error is found in the input parameters, the output directory `destination_folder/` is created. If it already exists and if `full_output` is set to 1, the user has the choice to overwrite the existing data or to stop the execution in order to choose another output folder, otherwise data will be overwritten nonetheless. For a subsequent data interpretation, the parameters file is copied in the output folder. The expected output at this stage if of the form:

```
          ############################
          #      BLACKHAWK v1.0      #
          #     HAWKING SPECTRUM     #
          #    COMPUTATION DEVICE    #
          ############################

[main] :  STARTING EXECUTION...
[main] :  READING THE RUN PARAMETERS IN 'parameters.txt'...       DONE
[main] :  ESTIMATION OF THE MEMORY USE...

          Running this session will use at least 60.668 MB of RAM
and 228.036 MB of disc memory.

          Do you want to continue?  (type y or n) y

                                                                    DONE
[main] :  SAVING RUN PARAMETERS...                                  DONE
```

The subsequent execution steps depend on the program. Output examples are given in the mode `full_output` $= 0$.

6.2 `BlackHawk_tot`: Full time-dependent Hawking spectra

In this program, `BlackHawk` computes the time-dependent Hawking spectra of a chosen initial distribution of BHs.

BlackHawk will compute the initial distribution of BHs (at tmin) using the routine spectrum or will read the user-defined BH distribution file table with the routine read_users_table (depending on the spectrum_choice), filling the arrays init_masses[], init_spins[] and spec_table[][]. It writes the results in the output with write_spectrum (see Section 5.2).

It then reads the $f(M, a^*)$ and $g(M, a^*)$ tables using the read_fM_table and read_gM_table routines, respectively, filling the arrays fM_table[][], gM_table[][], fM_masses[] and fM_a[], in order to evolve in time each initial BH spin and mass down to the Planck mass limit using the routine life_evolution. This fills the arrays life_times[], life_masses[][][], life_spins[][][] and dts[]. The evolutions in time are written in the output using the routine write_life_evolutions (see Section 5.3).

Then BlackHawk reads the greybody factor tables using the read_gamma_tables routine, filling the arrays gammas[][][], gamma_a[] and gamma_x[], and the fit tables using read_asymp_fits, filling the array fits[][][]. The common time range times[] is filled with the times in life_times[] until the evaporation of the last BH. This time range thus embeds all interesting intermediate evolution timesteps.

If the parameter primary_only has been set to 0, BlackHawk reads the suitable hadronization tables (depending on the hadronization_choice) with the routine read_hadronization_tables, filling the arrays tables[][][][], initial_energies[] and final_energies[]. It uses all these tables to compute the primary and secondary (if primary_only = 0) Hawking spectra using the routine total_spectra. Due to the large number of intermediate timesteps when a full distribution is considered, we do not perform the full computation in one step in the RAM memory, but rather do it timestep after timestep using the intermediate arrays partial_primary_spectra[][], partial_hadronized_spectra[][][] and partial_integrated_hadronized_spectra[][], and the instantaneous routines hadronize_instantaneous, integrate_initial_energies_instantaneous and add_*_instantaneous. The intermediate results are written in the output using write_lines (see Section 5.5).

This is the end of the execution of BlackHawk_tot. The expected output is of the form:

```
[main] :  COMPUTING THE INITIAL DISTRIBUTION OF BLACK HOLES...    DONE
[main] :  WRITING INTO FILE 'BH_spectrum.txt'...                  DONE
[main] :  READING EVOLUTION TABLES...                             DONE
[main] :  COMPUTING THE EVOLUTION OF BLACK HOLES...               DONE
[main] :  WRITING INTO FILE 'life_evolutions.txt'...              DONE
[main] :  READING GAMMA TABLES...                                 DONE
[main] :  READING FIT TABLES...                                   DONE
```

```
[main] :  READING HADRONIZATION TABLES...                    DONE
[main] :  COMPUTING SPECTRA...                               DONE
[main] :  END OF EXECUTION
```

6.3 `BlackHawk_inst`: Instantaneous Hawking spectra

In this program, `BlackHawk` computes the instantaneous Hawking spectra of a distribution of BHs.

First `BlackHawk` will compute the initial distribution of BHs (at `tmin`) using the routine `spectrum` or it will read the user-defined BH distribution file `table` with the routine `read_users_table` (depending on the `spectrum_choice`), filling the arrays `init_masses[]`, `init_spins[]` and `spec_table[][]`. It then writes the results in the output with `write_spectrum` (see Section 5.2).

Then `BlackHawk` reads the greybody factor tables using the routine `read_gamma_tables`, filling the arrays `gammas[][][]`, `gamma_masses[]` and `gamma_energies[]` and the fit table with the routine `read_asymp_fits`, filling the array `fits[][][]`, to compute the primary Hawking spectra using the routine `instantaneous_primary_spectrum`, filling the arrays `instantaneous_primary_spectra[][]`. The results are written in the output by the routine `write_instantaneous_primary_spectra` (see Section 5.4).

If the parameter `primary_only` has been set to 0, `BlackHawk` reads the hadronization tables (depending on the `hadronization_choice`) using the routine `read_hadronization_tables`, filling the arrays `tables[][][][]`, `initial_energies[]` and `final_energies[]`, and uses them to compute the secondary Hawking spectra using the routine `hadronize_instantaneous`, filling the array `instantaneous_hadronized _spectra[][][]`.

The initial energy dependence of the spectra is integrated out with the routine `integrate_initial_energies_instantaneous`, which fills the array `instantaneous_integrated_hadronized_spectra[][]`. The contributions from primary photons, neutrinos and electrons are added to the secondary spectra by the routines `add_*_instantaneous`. The results are written in the output by the routine `write_instantaneous_hadronized_spectra` (see Section 5.5).

This is the end of the execution of `BlackHawk_inst`. The expected output is of the form:

```
[main] :  COMPUTING THE INITIAL DISTRIBUTION OF BLACK HOLES...   DONE
[main] :  WRITING INTO FILE 'BH_spectrum.txt'...                 DONE
[main] :  READING GAMMA TABLES...                                DONE
[main] :  READING FIT TABLES...                                  DONE
[main] :  COMPUTING PRIMARY SPECTRA...                           DONE
[main] :  WRITING INTO FILE 'instantaneous_primary_spectra.txt'...DONE
[main] :  READING HADRONIZATION TABLES...                        DONE
```

```
[main] :  HADRONIZING PARTICLES...                                  DONE
[main] :  INTEGRATING OVER INITIAL ENERGIES...                      DONE
[main] :  WRITING INTO FILE 'instantaneous_secondary_spectra.txt'...DONE
[main] :  END OF EXECUTION
```

7 Output files

As explained in the previous sections, all the output files generated by a run of `BlackHawk` will be stored in `destination_folder/`. In this Section we describe the format of the files created by each program. Examples of results can be found in Appendix D and in [47–49]. In all cases, the parameter file `parameters.txt` used for the run is copied in the output folder in order to allow for subsequent data interpretation.

`Python` vizualisation scripts have been provided in the sub-folder `scripts/` in order to plot the data produced by both programs. They come with a `README` that explains how to configure them. The user can of course modify these scripts or use any other plotting program.

7.1 `BlackHawk_tot`

Running `BlackHawk_tot` produces four (or three if `primary_only` is set to 1) types of output files:

- `BH_spectrum.txt`: this file is written by the routine `write_spectrum`. It contains the initial density spectrum of BHs and is 2-dimensional: the first column is a list of the BH initial masses (in g), the first line a list of the initial reduced spins (dimensionless) and the bulk comoving number densities (in cm^{-3}).

- `life_evolutions.txt`: this file is written by `write_life_evolutions`. It contains all the integrated timesteps for each initial BH mass and spin. It includes the total number of integration timesteps. It also contains `BHnumber` tables in which the first column gives the time (in s), and each other pair of columns is the evolution of the mass (in g) and spin of BHs as a function of time for a fixed initial mass.

- `*_primary_spectrum.txt`: these files are written by the routine `write_lines`. They contain the emission rates of each primary particle at each final time and for each simulated initial energy. The first line gives the list of energies (in GeV), the first column gives the list of times (in s), and each further column is the emission rate of the particle per unit energy, time and covolume (in $\mathrm{GeV}^{-1}\cdot\mathrm{s}^{-1}\cdot\mathrm{cm}^{-3}$).

- `*_secondary_spectrum.txt`: these files are also written by `write_lines`. They contain the emission rates of each secondary particles at each final times and for each simulated final energies. The first line gives the list of energies (in GeV), the first column gives the list of times (in s), and each other column is the emission rate of the particle per units of energy, time and covolume (in $\mathrm{GeV}^{-1}\cdot\mathrm{s}^{-1}\cdot\mathrm{cm}^{-3}$). These files will not be generated if the parameter `primary_only` has been set to 1.

7.2 `BlackHawk_inst`

Running `BlackHawk_inst` produces three (or two if `primary_only` is set to 1) output files:

- `BH_spectrum.txt`: this file is written by the routine `write_spectrum`. It contains the initial density spectrum of BHs and is 2-dimensional: the first column is a list of the BH initial masses (in g), the first line a list of the initial reduced spins (dimensionless) and the bulk comoving number densities (in cm^{-3}).

- `instantaneous_primary_spectra.txt`: this file is written by the routine `write_instantaneous_primary_spectra`. It contains the emission rates of the primary particles for each simulated initial energy. The first line is the list of primary particles, the first column is the list of energies (in GeV), and each other column is the emission rate per unit energy and time (in $\mathrm{GeV}^{-1}\cdot\mathrm{s}^{-1}\cdot\mathrm{cm}^{-3}$).

- `instantaneous_secondary_spectra.txt`: this file is written by the routine `write_instantaneous_hadronized_spectra`. It contains the emission rates of the secondary particles for each simulated final energy. The first line is the list of secondary particles, the first column is that of energies, and each other column is the emission rate per unit energy and time (in $\mathrm{GeV}^{-1}\cdot\mathrm{s}^{-1}\cdot\mathrm{cm}^{-3}$). It will not be generated if the parameter `primary_only` has been set to 1.

8 Memory usage

The code `BlackHawk` has been designed to minimize the memory used (both RAM and disk) and the computation time while avoiding excessive approximations. In this Section we give estimates of the memory used by each program.

8.1 RAM used

To every array defined in `BlackHawk`, a memory space is allocated with a `malloc` call. This memory is freed at the moment the array stops being necessary for the subsequent part of the run. Then, the RAM used by `BlackHawk` at a given step of a session (corresponding to a paragraph in Section 6) can be estimated as a sum over

all active arrays at that time. `double` numbers are coded in 8 bytes and `int` in 4 bytes. Memory spaces M are given in bytes. For `BlackHawk_tot` we have:

- step 1 (BH spectrum):
  - `init_masses[]` $= 8 \times$ `BHnumber`,
  - `init_spins[]` $= 8 \times$ `BHnumber`,
  - `spec_table[]` $= 8 \times$ `BHnumber` $\times$ `anumber`,
- step 2 (BH evolution):
  - `init_masses[]` $= 8 \times$ `BHnumber`,
  - `init_spins[]` $= 8 \times$ `BHnumber`,
  - `spec_table[]` $= 8 \times$ `BHnumber` $\times$ `anumber`,
  - `fM_table[][]` $= 8 \times$ `nb_fM_a` $\times$ `nb_fM_masses`,
  - `gM_table[][]` $= 8 \times$ `nb_fM_a` $\times$ `nb_fM_masses`,
  - `fM_masses[]` $= 8 \times$ `nb_fM_masses`,
  - `fM_a[]` $= 8 \times$ `nb_fM_a`,
  - `life_masses[][][]` $= 8 \times$ `BHnumber`$^2 \times$ `anumber`$^2 \times$ `limit`,
  - `life_spins[][][]` $= 8 \times$ `BHnumber`$^2 \times$ `anumber`$^2 \times$ `limit`,
  - `life_times[]` $= 8 \times$ `BHnumber` $\times$ `anumber` $\times$ `limit`,
  - `dts[]` $= 8 \times$ `BHnumber` $\times$ `anumber` $\times$ `limit`,
- step 3 (primary and secondary spectra):
  - `spec_table[]` $= 8 \times$ `BHnumber` $\times$ `anumber`,
  - `life_masses[][][]` $= 8 \times$ `BHnumber`$^2 \times$ `anumber`$^2 \times$ `limit`,
  - `life_spins[][][]` $= 8 \times$ `BHnumber`$^2 \times$ `anumber`$^2 \times$ `limit`,
  - `life_times[]` $= 8 \times$ `BHnumber` $\times$ `anumber` $\times$ `limit`,
  - `dts[]` $= 8 \times$ `BHnumber` $\times$ `anumber` $\times$ `limit`,
  - `gammas[][][]` $= 8 \times 4 \times$ `nb_gamma_a` $\times$ `nb_gamma_x`,
  - `gamma_a[]` $= 8 \times$ `nb_gamma_a`,
  - `gamma_x[]` $= 8 \times$ `nb_gamma_x`,
  - `fits[][][]` $= 8 \times 4 \times$ `nb_gamma_a` $\times 7$,
  - `dof[]` $= 8 \times$ (`particle_number` + `grav`),
  - `spins[]` $= 8 \times$ (`particle_number` + `grav`),

- masses_primary[] = 8 × (particle_number + grav),
- times[] ≈ 8 × 1000 × BHnumber × anumber where an average of 1000 time iterations for the mass integration per BH has been assumed,
- energies[] = 8 × Enumber,
- tables[][][][] = 8×nb_fin_part×nb_init_en×nb_fin_en×nb_fin_part,
- initial_energies[] = 8 × nb_init_en,
- final_energies[] = 8 × nb_fin_en,
- partial_hadronized_spectra[][][] = 8 × nb_fin_part × Enumber × nb_fin_en,
- partial_primary_spectra[][] = 8×(particle_number+grav)×Enumber,
- partial_integrated_hadronized_spectra[][] = 8 × nb_fin_part × nb_fin_en,
- masses_secondary[] = 8 × nb_fin_part.

Using the parameters of Appendix D.1, the arrays occupy at most ∼ 150 MB.

For BlackHawk_inst we have:

- step 1 (BH spectrum):
  - BH_masses[] = 8 × BHnumber,
  - BH_spins[] = 8 × anumber,
  - spec_table[][] = 8 × BHnumber × anumber,
- step 2 (primary spectra):
  - BH_masses[] = 8 × BHnumber,
  - BH_spins[] = 8 × BHnumber,
  - spec_table[][] = 8 × BHnumber × anumber,
  - gammas[][][] = 8 × 4 × nb_gamma_a × nb_gamma_x,
  - gamma_a[] = 8 × nb_gamma_a,
  - gamma_x[] = 8 × nb_gamma_x,
  - fits[][][] = 8 × 4 × nb_gamma_a × 7,
  - dof[] = 8 × (particle_number + grav),
  - spins[] = 8 × (particle_number + grav),
  - masses_primary[] = 8 × (particle_number + grav),
  - instantaneous_primary_spectra[][] = 8×(particle_number+grav)× Enumber,

- `energies[]` $= 8 \times$ `Enumber`,

- step 3 (during hadronization):
  - `instantaneous_primary_spectra[][]` $= 8\times($`particle_number`$+$`grav`$)\times$ `Enumber`,
  - `energies[]` $= 8 \times$ `Enumber`,
  - `tables[][][][]` $= 8\times$`nb_fin_part`$\times$`nb_init_en`$\times$`nb_fin_en`$\times$`nb_fin_part`,
  - `initial_energies[]` $= 8 \times$ `nb_init_en`,
  - `final_energies[]` $= 8 \times$ `nb_fin_en`,
  - `masses_secondary[]` $= 8 \times$ `nb_fin_part`,
  - `instantaneous_hadronized_spectra[][][]` $= 8\times$`nb_fin_part`$\times$`Enumber`$\times$ `nb_fin_en`,

- step 3 bis (during integration):
  - `instantaneous_primary_spectra[][]` $= 8\times($`particle_number`$+$`grav`$)\times$ `Enumber`,
  - `energies[]` $= 8 \times$ `Enumber`,
  - `initial_energies[]` $= 8 \times$ `nb_init_en`,
  - `final_energies[]` $= 8 \times$ `nb_fin_en`,
  - `instantaneous_hadronized_spectra[][][]` $= 8\times$`nb_fin_times`$\times$`Enumber`$\times$ `nb_fin_en`,
  - `instantaneous_integrated_hadronized_spectra[][]` $= 8\times$`nb_fin_part`$\times$ `nb_fin_en`.

Using the parameters of Appendix D.1, the arrays occupy at most $\sim 10\,\mathrm{MB}$.

8.2 Static disk memory used

The output generated is written in `.txt` files using a precision of 5 significant digits. Adding the exponent and the coma, we obtain 12 characters per written number, which is 12 bytes. For `BlackHawk_tot` we have:

- file `BH_spectrum.txt`: $M = 12 \times ($`BHnumber` $+ 1) \times ($`anumber` $+ 1)$,
- file `life_evolutions.txt`: $M \approx 12 \times$ `BHnumber` $\times$ $(1. + 2 \times$ `anumber`$) \times$ `BHnumber` $\times$ `anumber` $\times 1000$ where an average number of 1000 time iterations for the mass integration per BH has been assumed,

- files `*_primary_spectrum.txt`: $M = 12 \times$ (`particle_number` + `grav`) $\times$ `Enumber` $\times\, 1000 \times$ `BHnumber` $\times$ `anumber` where an average number of 1000 time iterations for the mass integration per BH has been assumed,
- files `*_secondary_spectrum.txt`: $M = 12 \times$ `nb_fin_part` $\times$ `nb_fin_en` $\times\, 1000 \times$ `BHnumber` $\times$ `anumber` where an average number of 1000 time iterations for the mass integration per BH has been assumed.

Using the parameters of Appendix D.1, the total written disk space is $\sim 230\,\mathrm{MB}$.

For `BlackHawk_inst` we have:

- file `BH_spectrum.txt`: $M = 12 \times 3 \times$ `BHnumber`
- file `instantaneous_primary_spectra.txt`: $M = 12 \times$ `Enumber` $\times$ (`particle_number` + `grav`),
- file `instantaneous_secondary_spectra.txt`: $M = 12 \times$ `nb_fin_en` $\times$ `nb_fin_part`.

Using the parameters of Appendix D.1, the total written disk space is $\sim 35\,\mathrm{kB}$.

9 Other applications

In this Section we present some hints on how to modify `BlackHawk`. Most of these modifications will require add-ons in the parameter files and thus a modification of the routine `read_params` and of the structure `struct param`.

9.1 Computing new numerical tables

The user may be interested in recomputing the tables described in Appendix B, either to have more entries or to compute them with different methods for comparison. The easiest way to add tables in `BlackHawk` would be:

- authorize the corresponding "choice" parameters to have other integer values,
- put the new tables in a new directory in the `src/tables/` subfolder,
- create a `infos.txt` information file and the corresponding `read_*_infos` routine or modify the existing ones,
- add a switch into the tables reading routines,
- make sure that the way tables are used in the routines will be compatible with the format of the new ones.

All the scripts used to compute the current tables are included in `BlackHawk` in the subfolder `scripts/` together with `README` files.

9.2 Using another BH mass and spin distribution

The user may be interested in testing its own BHs distribution. Here are the main steps to add a pre-built distribution:

- add a "choice" parameter to the `struct param` choosing the distribution,
- add the corresponding analytical formula to the routines `M_dist` and `a_dist`,
- modify the parameter `tmin` if the distribution is valid at a different initial time.

Providing a tabulated initial distribution to `BlackHawk` is done by switching the parameter `spectrum_choice` to -1, putting the table file in the subfolder `users_spectra/` and giving its full file name (including the extension) to the parameter `table`. The format has to be:

- the first column for BH masses M, the first line for BH spins a^* and the bulk of the table for the comoving number densities $\mathrm{d}n(M, a^*)$ (with $\mathrm{d}M$ taken around M and $\mathrm{d}a^*$ taken around a^*),
- masses and densities in CGS units (g and cm^{-3} respectively), spins in dimensionless form,
- numbers in standard scientific notation,
- only additional text: a string of characters on the up-left corner (e.g. "mass/spin").

9.3 Adding primary particles

If the user wants to add hypothetical primary Hawking particles, the following steps have to be undertaken:

- increase the parameter `particle_number` in the greybody factor table information file `infos.txt` or add the new particle(s) with a switch similar to the one for the graviton,
- recompute the $f(M, a^*)$ and $g(M, a^*)$ tables to account for this(ese) new emission(s),
- if the spin(s) of the new particle(s) is(are) not among the greybody factor tables, compute the new ones and modify the `read_gamma_tables` and `read_gamma_fits` routines,
- add the new particle(s) to all the fixed length arrays of particle types (for example the file names or columns in the writing routines),
- eventually add its(their) contribution(s) to the secondary spectra.

9.4 Adding secondary particles

In order to add secondary Hawking particles to the code, one has to:

- recompute the hadronization tables to take new branching ratios into account,
- add the new particle(s) to all the fixed length arrays of particle types (for example the file names or columns in the writing routines),
- add the corresponding contribution(s) to the routine `contribution_instantaneous`.

9.5 Other types of Black Holes

If the user wants to compute the Hawking emission of BHs different from the Schwarzschild or Kerr ones, several ingredients are needed:

- add a switch to the parameter file to select amongst the new types of BHs,
- modify/add the Hawking temperature function `temp_BH` for these BHs,
- modify/add evolution routines `loss_rate_*` and `life_evolution` (e.g. for charged BHs a routine `loss_rate_Q` for the evolution of the charge parameter Q),
- compute the corresponding f, g and eventually new evolution parameters tables and add the corresponding reading routines (e.g. for charged BHs a routine `read_hM_table` to read the $q(M, a^*, Q)$ table where q would describe the evolution of the electric charge Q) and associated information in the `infos.txt` file,
- compute the new greybody factor tables and update the corresponding reading and interpolating routines `read_gamma_tables`, `read_gamma_fits` and `dNdtdE`.

Depending on the complexity of the BH model, the user may need to implement some or all of the above modifications.

10 Conclusion

`BlackHawk` is the first public code generating both primary and secondary Hawking evaporation spectra for any distribution of Schwarzschild and Kerr Black Holes, and their evolution in time. The primary spectra are obtained using greybody factors, and the secondary ones result from the decay and hadronization of the primary particles. The Black Holes and spectra evolution are obtained by considering the energy and angular momentum losses via Hawking radiation and the modification of

the temperature of the Black Hole. `BlackHawk` is designed in a user-friendly way and modifications can be easily implemented. The primary application is to study the effects of particles generated by Hawking evaporation on observable quantities and thus to disqualify or set constraints on cosmological models implying the formation of Black Holes, as well as to test the Hawking radiation assumptions and study Black Hole general properties.

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A Units

The `BlackHawk` code uses the GeV unit internally in order to have simpler analytical expressions. However, to make the user interface more accessible, the input parameters as well as the output files are in CGS units. We provide below unit conversions from the natural system of units where $\hbar = c = k_{\rm B} = G = 1$ to CGS or SI.

A.1 Energy

The energy conversion from GeV to Joules is:

$$E_{\rm J} = 1.602176565 \times 10^{-10}\, E_{\rm GeV}\,. \tag{A.1}$$

A.2 Mass

The dimensional link between energy and mass is $[m] = [E/c^2]$, and the conversion from GeV to grams is:

$$m_{\rm g} = 5.60958884 \times 10^{23}\, m_{\rm GeV}\,. \tag{A.2}$$

A.3 Time

The dimensional link between energy and time is $[t] = [\hbar/E]$, and the conversion from GeV to seconds is:

$$t_{\rm s} = 1.519267407 \times 10^{24}\, t_{\rm GeV^{-1}}\,. \tag{A.3}$$

A.4 Distance

The dimensional link between energy and distance is $[l] = [\hbar c/E]$, and the conversion from GeV to meters is:

$$l_{\rm cm} = 5.06773058 \times 10^{13}\, l_{\rm GeV^{-1}}\,. \tag{A.4}$$

A.5 Temperature

The dimensional link between energy and temperature is $[T] = [E/k_{\rm B}]$, and the conversion from GeV to Kelvins is:

$$T_{\rm K} = 8.61733063 \times 10^{-14}\, T_{\rm GeV}\,. \tag{A.5}$$

B Computation of the tables

B.1 Greybody factors

Chandrasekhar and Detweiler have shown that the Teukolsky equation can be reduced to a wave equation for Kerr Black Holes [50–53]. They found necessary to define a modified Eddington-Finkelstein radial coordinate r^* by the equation:

$$\frac{{\rm d}r^*}{{\rm d}r} = \frac{\rho^2}{\Delta}\,, \tag{B.1}$$

where $\rho(r)^2 \equiv r^2+\alpha^2$ and $\alpha^2 \equiv a^2+am/E$, a being the BH spin and m the projection of the angular momentum l. This equation can be integrated to give:

$$r^*(r) = r + \frac{r_{\rm H} r_+ + am/E}{r_+ - r_-} \ln\left(\frac{r}{r_+} - 1\right) - \frac{r_{\rm H} r_- + am/E}{r_+ - r_-} \ln\left(\frac{r}{r_-} - 1\right) , \tag{B.2}$$

where $r_{\rm H}$ is the BH Schwarzschild radius. Unfortunately, the inverse of this equation has to be found numerically and is generally difficult to determine with accurate precision. The Schrödinger-like wave equation is for all spins:

$$\frac{{\rm d}^2\psi_s}{{\rm d}r^{*2}} + \Big(E^2 - V_s(r^*)\Big)\,\psi_s = 0\,. \tag{B.3}$$

The method to transform Eq. (2.19) into this simple wave equation was proposed in the Chandrasekhar and Detweiler papers [50–53]. It is indeed difficult to find short-range potentials allowing for precise numerical computations. They give the form of such potentials in [50, 51] for spin 2, [52] for spins 0 and 1 and [53] for spin 1/2, The potentials are[8]:

$$V_0(r) = \frac{\Delta}{\rho^4}\left(\lambda_{0\,lm} + \frac{\Delta + 2r(r-M)}{\rho^2} - \frac{3r^2\Delta}{\rho^4}\right) , \tag{B.4}$$

$$V_{1/2,\pm}(r) = (\lambda_{1/2\,lm}+1)\frac{\Delta}{\rho^4} \mp \frac{\sqrt{(\lambda_{1/2,l,m}+1)\Delta}}{\rho^4}\left((r-M) - \frac{2r\Delta}{\rho^2}\right) , \tag{B.5}$$

$$V_{1,\pm}(r) = \frac{\Delta}{\rho^4}\left((\lambda_{1\,lm}+2) - \alpha^2\frac{\Delta}{\rho^4} \mp i\alpha\rho^2\frac{\rm d}{{\rm d}r}\left(\frac{\Delta}{\rho^4}\right)\right) , \tag{B.6}$$

$$\begin{aligned} V_2(r) = \frac{\Delta}{\rho^8}\Bigg(q - \frac{\rho^2}{(q-\beta\Delta)^2}\Big((q-\beta\Delta)\left(\rho^2\Delta q'' - 2\rho^2 q - 2r(q'\Delta - q\Delta')\right) \\ + \rho^2(\kappa\rho^2 - q' + \beta\Delta')(q'\Delta - q\Delta')\Big)\Bigg) . \end{aligned} \tag{B.7}$$

The different potentials for a given spin lead to the same results. In the potential for spin 2 particles, the following quantities appear:

$$q(r) = \nu\rho^4 + 3\rho^2(r^2-a^2) - 3r^2\Delta\,, \tag{B.8}$$

$$q'(r) = r\left((4\nu+6)\rho^2 - 6(r^2 - 3Mr + 2a^2)\right) , \tag{B.9}$$

$$q''(r) = (4\nu+6)\rho^2 + 8\nu r^2 - 6r^2 + 36Mr - 12a^2\,, \tag{B.10}$$

$$q'\Delta - q\Delta' = -2(r-M)\nu\rho^4 + 2\rho^2(2\nu r\Delta - 3M(r^2+a^2) + 6ra^2) + 12r\Delta(Mr - a^2)\,, \tag{B.11}$$

$$\beta_\pm = \pm 3\alpha^2\,, \tag{B.12}$$

$$\kappa_\pm = \pm\sqrt{36M^2 - 2\nu(\alpha^2(5\nu+6) - 12a^2) + 2\beta\nu(\nu+2)}\,, \tag{B.13}$$

[8] We found that the spin 0 potential had a missing "r" in [52].

$$q - \beta_+ \Delta = \rho^2(\nu\rho^2 + 6Mr - 6a^2)\,, \tag{B.14}$$
$$q - \beta_- \Delta = \nu\rho^4 + 6r^2(\alpha^2 - a^2) + 6Mr(r^2 - \alpha^2)\,, \tag{B.15}$$

where $\nu \equiv \lambda_{2\,lm} + 4$. As boundary conditions to solve Eq. (B.3), we use a purely ingoing wave. The solution at the horizon has the form:

$$\psi_s = e^{iEr^*}\,. \tag{B.16}$$

At infinity, the solution has the form:

$$\psi_s = A_{\rm in} e^{iEr^*} + A_{\rm out} e^{-iEr^*}\,. \tag{B.17}$$

In the Schwarzschild limit ($a = 0$), we recover the Regge-Wheeler potentials [54]. As the angular momentum projection m only appears multiplied by a, in this case the calculation is simplified since only one common value for all m has to be chosen once l is fixed. The sum over m thus reduces to a $l(l+1)$ factor and the $r(r^*)$ relation Eq. (B.1) is analytical.

The r^* variable change used in these potentials leads to divergences in the potentials, when $r^2 = r_{\rm div}^2 \equiv -\alpha^2$. This can happen for sufficiently low energies and high (negative) angular momentum projections, and it corresponds to the *superradiance* regime (for a very good review on the topic, see [55]). As discussed in the Chandrasekhar and Detweiler papers, the technique to avoid this divergence is to integrate Eq. (B.3) up to slightly before the divergence (e.g. $r_{\rm div} - \epsilon$). At this point, the behaviour of the potential V_s is known, and Eq. (B.3) is simplified, and the asymptotic form of the function ψ_s can be obtained for $\epsilon \to 0$. By continuity of the function R_s of Eq. (2.19) one can extrapolate this form up to slightly *after* the divergence (e.g. $r_{\rm div} + \epsilon$) and continue the integration.

Another difficulty which can arise is the fact that there can be an additional divergence in the spin 2 potential because of the $q - \beta_\pm \Delta$ term. For this extra divergence, we try to integrate with one of the potentials (e.g. κ_+, β_+), and in case of problem we try with the other potentials (e.g. κ_+, β_-), as it seems that at least one of the four combinations does not generate any divergence.

The greybody factor is given by the transmission coefficient of the wave from the horizon to the infinity:

$$\Gamma_{slm} \equiv T_{slm} = \frac{1}{|A_{\rm in}|^2}\,. \tag{B.18}$$

Practically, we compute the value of a single helicity/color d.o.f. emissivity:

$$Q_s \equiv \sum_{l,m} \frac{\Gamma_{slm}}{(e^{\omega'/T} \pm 1)}\,, \tag{B.19}$$

for some values of $0 \leq a^* \leq 0.9999$ and for a range of $0.01 \leq x \equiv 2Er_{\rm H} \leq 5$ (dimensionless), since we can show that these are the only relevant parameters for

massless particles. For x out of this range, we have found easier to fit empiric asymptotic forms to the emissivities. At low energies, we have for all spins:

$$\log_{10}(Q_s) \approx a_{1,s} \log_{10}(x) + a_{2,s} \,, \tag{B.20}$$

and at high energies:

$$\log_{10}(Q_s) \approx a_{3,s} x + a_{4,s} + a_{5,s} \cos(a_{7,s} x) + a_{6,s} \sin(a_{7,s} x) \,. \tag{B.21}$$

We fitted the computed emissivities to find the values of the parameters $a_{i,s}$. We checked that they agree with the asymptotic limits of [20] in the Schwarzschild case and of [56] in the Kerr case.

The `Mathematica` scripts `spin_*.m`, the fitting script `exploitation.m` as well as a `C` formating script `formating.c` and a `README` are provided in the subfolder:

`scripts/greybody_scripts/greybody_factors/`

If these tables are recomputed, it is advisable to also modify the information in the file:

`src/tables/gamma_tables/infos.txt`

B.2 Evolution tables

To compute the integrals of Eqs. (2.26) and (2.27), we use the greybody factor tables and the fits computed in Appendix B.1. The peak of Hawking emission lies around the BH temperature (see [20] for example), thus the integral does not need to be computed over all energies, but a restrained set $10^{-5} \times T < E < 10^{5} \times T$ is sufficient. The domains of integration are segmented over logarithmically distributed energies, and computed for masses between $M_{\rm P}$ to $10^{46}\,$GeV ($\sim 10^{-5} - 10^{22}\,$g). We compute these tables with and without the graviton emission. In the limit where neutrinos are massless, $f(M, a^*)$ and $g(M, a^*)$ are not expected to change for masses higher than $10^{22}\,$g, the tables can therefore be extended manually without any new computation. Masses are given in GeV (corresponding to grams) and $f(M, a^*)$ and $g(M, a^*)$ are in GeV^4 (corresponding to g^3·s^{-1} and g^2·GeV·s^{-1}, respectively). We have checked that the value of $f(M, 0)$ is consistent with that of [40] in the Schwarzschild case and that the values of $f(M, a^*)$ and $g(M, a^*)$ are consistent with [57] in the Kerr case.

The `C` script `fM.c` used to compute the tables and a `README` are provided in the subfolder:

`scripts/greybody_scripts/fM/`

If there tables are recomputed, it is advisable to also modify the information in the file:

`src/tables/fM_tables/infos.txt`

B.3 Hadronization

Two particle physics codes have been used to compute hadronization tables: `HERWIG` [42] and `PYTHIA` [43]. In both cases, the strategy is to generate the output of a collision (for example $e^+ + e^- \to u + \overline{u} \to ...$), and then to count the number of final particles (here denoted as dots) normalized by the number (here 2) of initial particles (here u, see Table 3 in Appendix C) satisfying the desired stability criterion: Table 4 for early Universe/BBN particles (`PYTHIA` and `HERWIG` tables) and Table 5 for present epoch particles (`PYTHIA` "new" tables). It gives the number of secondary Hawking particles of each type that a primary particle will generate.

To build the `PYTHIA` and `HERWIG` tables, we have simulated for each channel listed in Table 2, 10^5 events for initial energies E' (half of the center of mass energy) logarithmically distributed between 5 GeV and 10^5 GeV (`PYTHIA` and `PYTHIA` "new") or between 25 GeV and 10^5 GeV (`HERWIG`). Then, the final particles have been listed as a function of their final energy E, into a range of 10^{-6} GeV to 10^5 GeV and the counts have been averaged over the number or simulated events. This gives the dimensionless quantities $\mathrm{d}N_j^i(E', E)$ of Eq. (2.29). Energies are given in GeV.

particle	`PYTHIA` (new)	`HERWIG`
gluons	$e^+e^- \to h^0 \to g\overline{g}$	$e^+e^- \to h^0 \to g\overline{g}$
Higgs boson	$e^+e^- \to h^0$	$e^+e^- \to h^0$
W bosons	$e^+e^- \to Z^0/\gamma^* \to W^+W^-$	$e^+e^- \to Z^0/\gamma^* \to W^+W^-$
Z boson	$e^+e^- \to h^0 \to Z^0Z^0$	$e^+e^- \to Z^0/\gamma^* \to Z^0Z^0$
leptons	$e^+e^- \to h^0 \to l^+l^-$	$e^+e^- \to Z^0/\gamma^* \to l^+l^-$
quarks	$e^+e^- \to Z^0/\gamma^* \to q\overline{q}$	$e^+e^- \to Z^0/\gamma^* \to q\overline{q}$

Table 2. List of the channels used to compute the hadronization tables.

The branching ratios $e^\pm \to \gamma\gamma \to ...$ and $e^\pm \to \nu\overline{\nu} \to ...$ have not been computed. The contribution from the primary photons, neutrinos and electrons are directly added to the secondary spectra with a branching ratio of 1.

For initial energies lower than the cutoff of the computed tables, branching ratios from the lowest relevant initial energy will be extrapolated at lower energies once shifted to the considered energy, taking into account that no emission can arise below the rest mass of the final particles. There is however no guarantee that the extrapolations remain valid far beyond the low energy cutoff.

The `PYTHIA` (new) and `HERWIG` scripts used to run the particle physics codes, as well as the `C` scripts `formating.c` used to format the hadronization tables and `README` files are provided in the subfolders:

`scripts/pythia_scripts/`

`scripts/herwig_scripts/`

`scripts/pythia_scripts_new/`

Please contact one of the authors if you have issues using these scripts. If these tables are recomputed, it is advisable to also modify the informations in the files:

`src/tables/pythia_tables/infos.txt`

`src/tables/pythia_nex_tables/infos.txt`

`src/tables/herwig_tables/infos.txt`

C Particle information

particle	symbol	mass (GeV/c^2)	spin	quantum d.o.f.
Higgs boson	h^0	1.2503×10^2	0	1
photon	γ	0	1	2
gluons	g	0	1	16
W bosons	$W^\pm$	8.0403×10^1	1	6
Z boson	Z^0	9.11876×10^1	1	3
neutrinos	$\nu_{e,\mu,\tau}, \overline{\nu}_{e,\mu,\tau}$	0	1/2	6
electron	$e^\pm$	$5.109989461 \times 10^{-4}$	1/2	4
muon	$\mu^\pm$	$1.056583745 \times 10^{-1}$	1/2	4
tau	$\tau^\pm$	1.77686	1/2	4
up quark	$u, \overline{u}$	2.2×10^{-3}	1/2	12
down quark	$d, \overline{d}$	4.7×10^{-3}	1/2	12
charm quark	$c, \overline{c}$	1.27	1/2	12
strange quark	$s, \overline{s}$	9.6×10^{-2}	1/2	12
top quark	$t, \overline{t}$	1.7321×10^2	1/2	12
bottom quark	$b, \overline{b}$	4.18	1/2	12
graviton	G	0	2	2

Table 3. Properties of the elementary particles of the Standard Model, in addition to the graviton [14]. The number of quantum d.o.f. is the product of the family, antiparticle, the colour and the helicity multiplicities. Neutrinos are here considered massless. In the code, gluons have been assigned an effective mass to account for the QCD energy scale $\Lambda \approx 200\,\text{MeV}$.

particle	symbol	lifetime (s)
photon	γ	∞
electron	$e^\pm$	∞
muon	$\mu^\pm$	$(2.1969811 \pm 0.0000022) \times 10^{-6}$
neutrinos	$\nu_{e,\mu,\tau}, \overline{\nu}_{e,\mu,\tau}$	∞
charged pions	$\pi^\pm$	$(2.6033 \pm 0.0005) \times 10^{-8}$
neutral "long" kaon	K^0_L	$(5.099 \pm 0.021) \times 10^{-8}$
charged kaons	$K^\pm$	$(1.2379 \pm 0.0021) \times 10^{-8}$
proton	p,$\overline{\text{p}}$	∞
neutron	n,$\overline{\text{n}}$	880.2 ± 1

Table 4. Particles with a lifetime longer than $10^{-8}\,\text{s}$, relevant for early Universe/BBN studies and used to compute the hadronization tables in `pythia_tables/` and `herwig_tables/`.

particle	symbol	lifetime (s)
photon	γ	∞
electron	$e^{\pm}$	∞
neutrinos	$\nu_{e,\mu,\tau}, \overline{\nu}_{e,\mu,\tau}$	∞
proton	p,$\overline{\text{p}}$	∞

Table 5. Stable particles, relevant for evaporating BH in the present Universe and used to compute the hadronization table in `pythia_tables_new/`.

D Results

The results in the output files are given in CGS units.

D.1 Parameters

An example of `parameters.txt` file is given here:

```
destination_folder = test
full_output = 0
interpolation_method = 0

BHnumber = 1
Mmin = 1.e+15
Mmax = 1.e+17
anumber = 1
amin = 0.
amax = 0.5

spectrum_choice = 0
spectrum_choice_a = 0

amplitude_lognormal = 1.
stand_dev_lognormal = 1.
crit_mass_lognormal = 1.

amplitude_powerlaw = 1.
eqstate_powerlaw = 0.3333

amplitude_critical_collapse = 1.
crit_mass_critical_collapse = 1.

amplitude_uniform = 1.
```

```
stand_dev_a_gaussian = 1.
mean_a_gaussian = 1.

table = table.txt

tmin = 1.e-30
limit = 5000

Enumber = 100
Emin = 5.
Emax = 1.e+5
particle_number = 15
grav = 1

primary_only = 0

hadronization_choice = 2
```

D.2 `BlackHawk_tot`

When running `BlackHawk_tot` with the parameters of Appendix D.1, the output file `BH_spectrum.txt` is generated in the folder `test/` and reads:

```
Initial BH comoving number density as a function of their mass and spin.

 mass/spin     0.00000e+000
 1.00000e+010  1.00000e+000
```

The output file `life_evolutions.txt` contains:

```
Evolution of the BH masses and spins as functions of time.
Total number of time iterations:  817
```

```
 t             M             a
 1.00000e-030  1.00000e+010  0.00000e+000
 2.00000e-030  1.00000e+010  0.00000e+000
 4.00000e-030  1.00000e+010  0.00000e+000
 8.00000e-030  1.00000e+010  0.00000e+000
 1.60000e-029  1.00000e+010  0.00000e+000
 3.20000e-029  1.00000e+010  0.00000e+000
 6.40000e-029  1.00000e+010  0.00000e+000
 1.28000e-028  1.00000e+010  0.00000e+000
 2.56000e-028  1.00000e+010  0.00000e+000
 5.12000e-028  1.00000e+010  0.00000e+000
 ...           ...           ...
```

The output file `photon_primary_spectrum.txt` reads:

```
Hawking primary spectrum as a function of time.
 time/energy   5.00000e+000  5.52605e+000  6.10744e+000  6.75001e+000  ...
 1.00000e-030  3.45624e+012  4.63136e+012  6.20602e+012  8.31607e+012  ...
 2.00000e-030  3.45624e+012  4.63136e+012  6.20602e+012  8.31607e+012  ...
 4.00000e-030  3.45624e+012  4.63136e+012  6.20602e+012  8.31607e+012  ...
 8.00000e-030  3.45624e+012  4.63136e+012  6.20602e+012  8.31607e+012  ...
 1.60000e-029  3.45624e+012  4.63136e+012  6.20602e+012  8.31607e+012  ...
 3.20000e-029  3.45624e+012  4.63136e+012  6.20602e+012  8.31607e+012  ...
 6.40000e-029  3.45624e+012  4.63136e+012  6.20602e+012  8.31607e+012  ...
 1.28000e-028  3.45624e+012  4.63136e+012  6.20602e+012  8.31607e+012  ...
 2.56000e-028  3.45624e+012  4.63136e+012  6.20602e+012  8.31607e+012  ...
 5.12000e-028  3.45624e+012  4.63136e+012  6.20602e+012  8.31607e+012  ...
 ...           ...           ...           ...           ...           ...
```

The output file `photon_secondary_spectrum.txt` contains:

```
Hawking secondary spectrum as a function of time.
```

```
time/energy   1.00000e-006  1.05200e-006  1.10700e-006  1.16400e-006  ...
1.00000e-030  2.58419e+031  2.47040e+031  2.37726e+031  2.24715e+031  ...
2.00000e-030  2.58419e+031  2.47040e+031  2.37726e+031  2.24715e+031  ...
4.00000e-030  2.58419e+031  2.47040e+031  2.37726e+031  2.24715e+031  ...
8.00000e-030  2.58419e+031  2.47040e+031  2.37726e+031  2.24715e+031  ...
1.60000e-029  2.58419e+031  2.47040e+031  2.37726e+031  2.24715e+031  ...
3.20000e-029  2.58419e+031  2.47040e+031  2.37726e+031  2.24715e+031  ...
6.40000e-029  2.58419e+031  2.47040e+031  2.37726e+031  2.24715e+031  ...
1.28000e-028  2.58419e+031  2.47040e+031  2.37726e+031  2.24715e+031  ...
2.56000e-028  2.58419e+031  2.47040e+031  2.37726e+031  2.24715e+031  ...
5.12000e-028  2.58419e+031  2.47040e+031  2.37726e+031  2.24715e+031  ...
...           ...           ...           ...           ...           ...
```

D.3 `BlackHawk_inst`

When running `BlackHawk_inst` with the parameters of Appendix D.1, the output file `instantaneous_primary_spectra.txt` is generated in the folder `test/` and reads:

```
Hawking primary spectra for each particle types.
 energy/particle  photon        gluons         higgs          W+-            ...
 5.00000e+000     3.45624e+012  2.76499e+013  0.00000e+000  0.00000e+000  ...
 5.52605e+000     4.63136e+012  3.70509e+013  0.00000e+000  0.00000e+000  ...
 6.10744e+000     6.20602e+012  4.96482e+013  0.00000e+000  0.00000e+000  ...
 6.75001e+000     8.31607e+012  6.65286e+013  0.00000e+000  0.00000e+000  ...
 7.46017e+000     1.11435e+013  8.91483e+013  0.00000e+000  0.00000e+000  ...
 8.24506e+000     1.49324e+013  1.19459e+014  0.00000e+000  0.00000e+000  ...
 9.11252e+000     2.00094e+013  1.60075e+014  0.00000e+000  0.00000e+000  ...
 1.00712e+001     2.68126e+013  2.14500e+014  0.00000e+000  0.00000e+000  ...
 1.11308e+001     3.59288e+013  2.87431e+014  0.00000e+000  0.00000e+000  ...
 1.23019e+001     4.81447e+013  3.85157e+014  0.00000e+000  0.00000e+000  ...
 ...              ...           ...           ...           ...           ...
```

The output file `instantaneous_secondary_spectra.txt` contains:

```
Hawking secondary spectra for each particle type.
```

energy/particle	photon	electron	nu_e	nu_mu	...
1.00000e-006	2.58419e+031	0.00000e+000	7.55869e+026	0.00000e+000	...
1.05200e-006	2.47040e+031	0.00000e+000	8.92620e+020	0.00000e+000	...
1.10700e-006	2.37726e+031	0.00000e+000	5.13357e+014	0.00000e+000	...
1.16400e-006	2.24715e+031	0.00000e+000	8.06556e+020	0.00000e+000	...
1.22500e-006	2.17158e+031	0.00000e+000	7.66555e+020	0.00000e+000	...
1.28900e-006	2.07013e+031	0.00000e+000	5.50633e+021	0.00000e+000	...
1.35600e-006	1.93968e+031	0.00000e+000	4.62106e+021	0.00000e+000	...
1.42700e-006	1.86141e+031	0.00000e+000	7.50867e+026	0.00000e+000	...
1.50100e-006	1.78117e+031	0.00000e+000	7.16266e+026	0.00000e+000	...
1.57900e-006	1.68781e+031	0.00000e+000	4.15153e+026	0.00000e+000	...
...	...	...	...	...	...

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